



Tuned Mass Damper for Reduction of Seismic Loads in High-Rise Residential Buildings

D. E. Bondarev^(✉)

Principal-Structural Engineering, CKTI-Vibroisem (CVS) Ltd., St. Petersburg, Russia
dbondarev@cvs.spb.su

Abstract. Tuned mass dampers (TMD) are used primarily for reduction of seismic and wind oscillations in high-rise buildings. It is well-known that the base isolation isn't effective in tall buildings. In general, TMD can reduce seismic loads in tall building but it needs a large mass of TMD. In addition, TMD can't reduce vertical oscillations, which can be very destructive due to P-delta effect. This paper presents an engineering solution for mitigation of response of structure caused by seismic excitations. An approach of using the upper part of the building as a TMD can significantly reduce horizontal accelerations and stresses in building elements up to 50% along the entire height. Also proposed TMD can significantly reduce vertical oscillations in a primary building up to 30% in comparison with building without TMD. This solution can be used both in existing and in new built buildings. This solution doesn't require any additional mass and its transportation to the installation place. Optimization criterion for defining optimal TMD's properties was developed. Criterion presents the aim's function of maximum difference between accelerations of floors with and without TMD along the entire height. For analytical studies matrix of stiffness and dissipation matrix were developed. Matrix of stiffness considers bending and sliding motions and dissipation matrix consider damping ratio for soil, TMD's constructions and constructions of the building.

Keywords: TMD · tuned mass damper · seismic load · vibration control · optimization

1 Introduction

Tuned mass damper (TMD) is a device for reduction of seismic and wind responses of buildings and structures. The first reference of usage appeared in 1909 when Frahm received a patent «Device for damping vibrations of bodies» [1]. This device is used in structures to prevent discomfort, damage or structural failure caused by dynamic excitations. Vibration control systems can be divided into four groups: active, semi-active, hybrid and passive systems [2]. It is possible to use several types of vibration control systems in one building [3]. TMD is a passive control device which doesn't require any external sources of energy. There are many types of TMDs: friction TMD [4], conventional TMD [5], pendular TMD [6], bidirectional [7], tuned liquid column

damper (TLCDs) [8] etc. TMDs are widely used in tall structures [9], chimneys [10], long span transmission tower-line systems [11], high-rise buildings [12, 13], flexible bridges [14] etc. Usually, TMDs are installed at the upper floors of high-rise building, under bridge's spans and/or at bridge pylons, etc.

In general, TMD consists of a mass, a spring, and a damper. A two-degree-of-freedom (DOF) system is shown in Fig. 1 and Fig. 2 for seismic and wind excitations respectively.

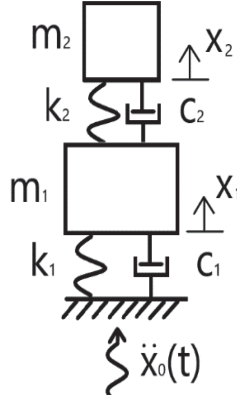


Fig. 1. A two DOF damped system subjected to seismic excitation

Applying Newton's second law to the main mass m_1 gives:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) = -m_1\ddot{x}_0 \quad (1)$$

Applying Newton's second law to the mass of a TMD m_2 gives:

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = -m_2\ddot{x}_0 \quad (2)$$

Transforming these equations, we receive a system of differential equations of the second order:

$$\begin{cases} \ddot{x}_1 = -\ddot{x}_0 - \frac{k_1}{m_1}x_1 - \gamma^2\mu\omega_1^2(x_1 - x_2) - 2\xi_2\omega_1\gamma\mu(\dot{x}_1 - \dot{x}_2) - 2\xi_1\omega_1\dot{x}_1 \\ \ddot{x}_2 = -\ddot{x}_0 - \omega_2^2(x_2 - x_1) - 2\xi_2\omega_1\gamma(\dot{x}_2 - \dot{x}_1) \end{cases} \quad (3)$$

Where m_1, k_1 are the mass, stiffness of a primary structure, m_2, k_2 are the mass, stiffness of a TMD's construction, \ddot{x}_0 – time history acceleration of seismic excitation. $x_1, x_2, \dot{x}_1, \dot{x}_2, \ddot{x}_1, \ddot{x}_2$ are displacements, velocities and accelerations of structure and the TMD, μ represents the ratio of the TMD mass (m_2) to structural mass (m_1). ξ_1, ξ_2 are the damping ratios of the structure and the TMD, γ is the ratio of the frequency of the TMD (ω_2) to the frequency of the structure (ω_1).

For the system subjected to wind excitation where $P(t)$ is a time-dependent external force:

$$\begin{cases} \ddot{x}_1 = \frac{P(t)}{m_1} - \frac{k_1}{m_1}x_1 - \gamma^2\mu\omega_1^2(x_1 - x_2) - 2\xi_2\omega_1\gamma\mu(\dot{x}_1 - \dot{x}_2) - 2\xi_1\omega_1\dot{x}_1 \\ \ddot{x}_2 = -\omega_2^2(x_2 - x_1) - 2\xi_2\omega_1\gamma(\dot{x}_2 - \dot{x}_1) \end{cases} \quad (4)$$

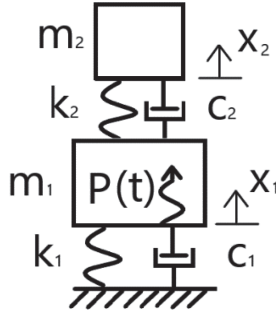


Fig. 2. A two DOF damped system subjected to wind excitation

These equations of motion undergoing seismic and wind excitations have an analytical solution if an external force (seismic or wind load) is roughly expressed by harmonic loading. For wind excitation it is a sinusoidal time-dependent pressure of wind and for seismic excitation a sinusoidal displacement of soil motion [15].

Also, these equations can be solved by numerical integration method. For instance, it can be solved by Runge-Kutta method where right parts in Eqs. (3), (4) are vectors of the first and the second derivatives in an explicit form and, in addition, it is necessary to use a vector of initial conditions such as velocity and displacement in the beginning of the motion. Using numerical solution of differential equations, it is possible to take into account all frequencies of the external force expressed with accelerograms and wind pressure time histories.

It is well-known that there are simple equations to define the optimum damping ratio and frequency (f_2 , Hz) of a TMD. For minimum structural displacement amplitude, the formulae were given by Den Hartog's [16]:

$$f_2 = \frac{f_1}{1 + \mu} \quad (5)$$

$$\xi_2 = \sqrt{\frac{3\mu}{8(1 + \mu)^3}} \quad (6)$$

If the acceleration amplitude of the structure is to be minimized:

$$f_2 = \frac{f_1}{\sqrt{1 + \mu}} \quad (7)$$

$$\xi_2 = \sqrt{\frac{3\mu}{4(2 + \mu)(1 + \mu)}} \quad (8)$$

There are many other criteria of optimization: maximum dynamic stiffness of the main structure [17], maximum effective damping of combined structure [18], minimum travel of damper mass relative to the main structure [18], minimum force in the main structure [19], minimum velocity of the main structure [19] etc. [15, 20, 21].

The Eqs. (3), (4) are very simple to use to show the main principles of working TMD. However, they can't be used in actual engineering practice because real civil engineering structures can't be considered as single-DOF systems. External excitation doesn't have the only frequency and structures may undergo nonlinear deformations. Also, it is necessary to consider the stiffness of soil in equations of motions if a structure is located at a soft soil.

In general, TMD shall be tuned close to a dominant response frequency of the structure. The TMD usually requires an essential mass related to a mass of a primary structure and a large space for its installation at high elevations. Usually ratio μ of a mass of TMD's construction and a mass of a primary construction of building ranges between 0.02 and 0.08 [22] for buildings subjected to seismic excitations and 0.0005 and 0.02 [23] for buildings subjected to wind excitations to achieve demanded TMD's efficiency.

TMDs are rather complex and expensive devices that are limited in mass and damping with levels far away from optimal parameters and usually are tuned to only one dominant frequency of the structure providing protection from wind loads only and being ineffective in case of seismic excitation. Inefficiency of the TMD in case of seismic excitation makes researchers find other technological and engineering solutions. And it is necessary to search other criterion of optimization for increasing efficiency of the TMD's system.

The TMD approach, that is described in this paper, allows significantly improve the TMD's efficiency against seismic excitation and create a three-dimensional TMD system with optimal mass, stiffness and damping parameters for structures' protection, while significantly reducing the cost of TMD itself.

2 Methods

2.1 The Motion Equation of a Multi-Degree-Of-Freedom (MDOF) System of Shear-Wall Building with the TMD

The motion equation of a MDOF system for the high-rise building subjected to a seismic excitation can be written as follows:

$$[M]\ddot{u} + [C]\dot{u} + [K]u = -[M](\{J_x\}\ddot{x}_0 + \{J_y\}\ddot{y}_0 + \{J_z\}\ddot{z}_0) \quad (9)$$

Where $[M]$, $[C]$ and $[K]$ represent the mass, damping and stiffness matrices, respectively, u , \dot{u} , \ddot{u} are the relative displacement, velocity, acceleration vectors with respect to the base. $\{J_x\}$, $\{J_y\}$, $\{J_z\}$ are the vectors which consist of cosines between vector of displacements and vector of excitation. \ddot{x}_0 , \ddot{y}_0 , \ddot{z}_0 – time history acceleration of a seismic excitation in X , Y , Z directions.

Considering only shear stiffness of the floors and X-direction of seismic excitation we may use the following equation:

$$[M]\ddot{x} + [C]\dot{x} + [K]x = -[M](\{J_x\}\ddot{x}_0) \quad (10)$$

MDOF system with TMD is shown in Fig. 3.

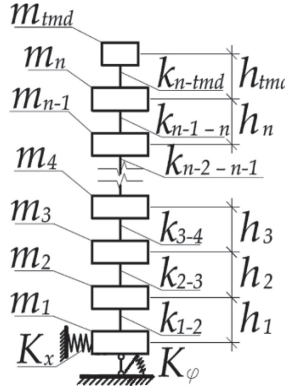


Fig. 3. MDOF system of shear-wall building subjected to a seismic excitation in X-direction with TMD located on the last floor

Stiffness matrix can be written as follows [24]:

$$[K] = \begin{bmatrix} K_x + K_{1-2} & -K_{1-2} & & \dots & 0 & & 0 & K_{1-2}h_1 \\ -K_{1-2} & K_{1-2} + K_{2-3} & & \dots & 0 & & 0 & (K_{2-3}h_2 - K_{1-2}h_1) \\ & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots \\ & 0 & 0 & & 0 & K_{n-1-n} + K_{TMD} & -K_{TMD} & (K_{TMD}h_{TMD} - K_{n-1-n}h_n) \\ 0 & & 0 & & 0 & -K_{TMD} & K_{TMD} & -K_{TMD}h_{TMD} \\ K_{1-2}h_1 & (K_{2-3}h_2 - K_{1-2}h_1) & \dots & (K_{TMD}h_{TMD} - K_{n-1-n}h_n) & -K_{TMD}h_{TMD} & \sum_{i=1}^n (K_{i,i+1}h_i^2 + K_\phi) \end{bmatrix} \quad (11)$$

Mass matrix [24]:

$$[M] = \begin{bmatrix} m_1 & \dots & \dots & 0 \\ \vdots & \ddots & 0 & \vdots \\ \vdots & 0 & m_n & \vdots \\ 0 & \dots & \dots & I \end{bmatrix}; I = I_c + \sum_{i=1}^n m_i h_i^2 \quad (12)$$

Where I_c is a moment of inertia of the construction relative to horizontal axes passing through the center of gravity (CG); h_i – is dimension between i -floor and CG; K_x , K_ϕ – translational and rocking stiffnesses of soil. $K_{i,i+1}$ – shear stiffness of the floor; K_{TMD} – stiffness of the TMD.

Vector $\{J_x\}$ will be:

$$\{J_x\}^T = \{1, \dots, 1, 0\} \quad (13)$$

Model in Fig. 3 can be used only in case of shear-wall buildings. It is necessary to develop stiffness matrix for considering shear and bending stiffnesses of a building.

$2123292340 \text{ t} * \text{m}^2$, $EI_X = 2550000000 \text{ t} * \text{m}^2$, translational and rocking stiffnesses of soil $K_x = 1576390 \text{ t/m}$, $K_\phi = 517761250 \text{ t} * \text{m}$, $\xi_{SOIL} = 0.15$. The eigen frequencies of the dynamically uncontrolled building are shown in Table 1. Seismic excitation in X, Y and Z directions are shown in Figs. 7, 8, 9 and 10.

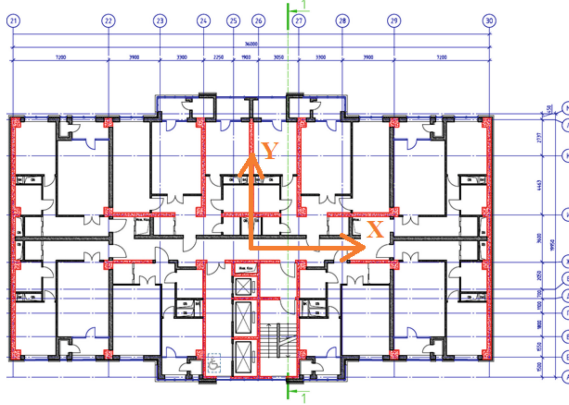


Fig. 6. Typical floor plan

Table 1. Eigen frequencies of dynamically uncontrolled building without TMD

Number	Circular frequency, rad/s	Frequency, Hz	Direction	Modal mass, %
1	1.99	0.32	Y	67.46
2	2.04	0.32	X	64.85
3	12.41	1.98	X	21.94
4	12.6	2.01	Y	21.08
5	32.22	5.13	X	9.01
6	33.63	5.35	Y	7.77
7	58.02	9.23	X	3.15
8	61.88	9.85	Y	2.7

In order to define stiffness, mass and damping ratio of the TMD it is necessary to carry out optimization analysis. Criterion of optimization:

$$Cr = \max \sum_{i=1}^n (\max|A_i| - \max|A_i^{TMD}|) \quad (18)$$

Where n – number of a floor (substructure), $\max|A_i|$ - maximum absolute accelerations of the n^{th} floor for the dynamically uncontrolled building (without TMD), $\max|A_i^{TMD}|$ - maximum absolute accelerations of the n^{th} floor for the dynamically controlled building (with TMD).

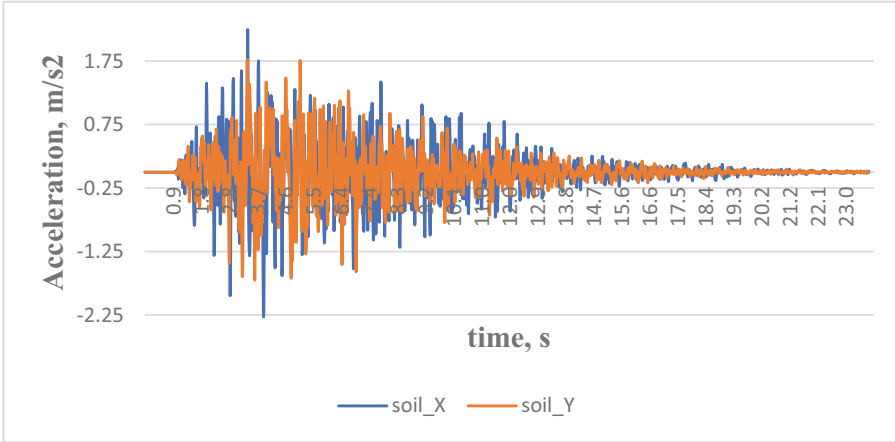


Fig. 7. Accelerations on soil in X, Y directions

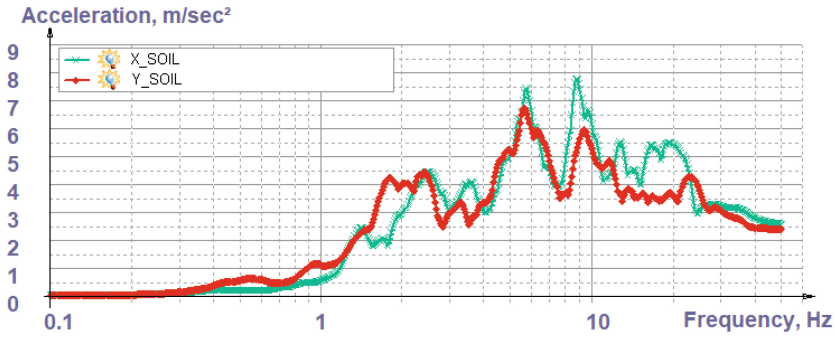


Fig. 8. Response spectra ($\xi = 0.05$) in X, Y directions

3 Results and Discussion

3.1 The Optimum Parameters of the TMD

The optimum parameters were defined by taking damping ratio of the TMD $\xi_{TMD} = 0.1$ (using Eq. (8)) for the first iteration of optimization, $M_{TMD} = 1050$ ton ($\mu = 2.3\%$) and using the model which is shown in Fig. 4 and consistently varying horizontal stiffness of the TMD. The acceleration along the entire height of the building is shown in Fig. 11, 12. Accelerations of the 13th floor in X and Y directions are shown in Figs. 13, 14, 15 and 16.

The optimum parameters for the TMD were installed as: $K_X = 12000$ t/m, $K_Y = 15000$ t/m, $\xi_{TMD} = 0.1$, $M_{TMD} = 1050$ ton. The TMD with these parameters is tuned to the 2nd eigen frequency of the building. During modal analysis in dissipation matrix (Eq. (16)) $\xi_1 = \xi_{soil}$, $\xi_2 = \xi_{TMD}$ were used. Installation of the TMD led to a minor changes in the eigen frequencies of the dynamically controlled building (Table 2).

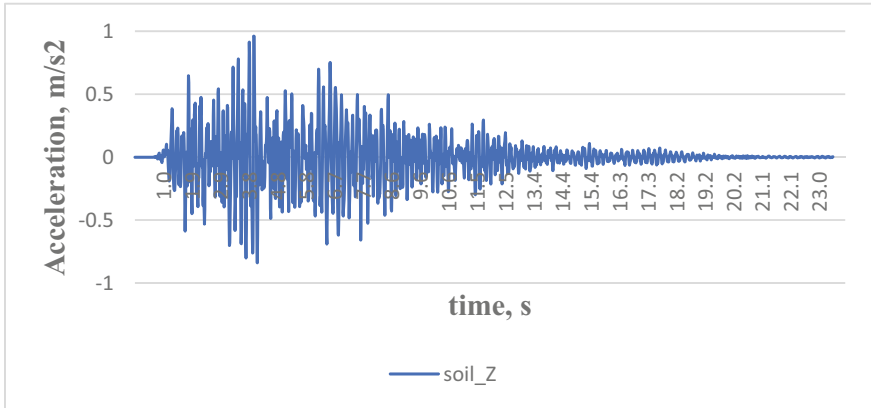


Fig. 9. Accelerations on soil in Z direction

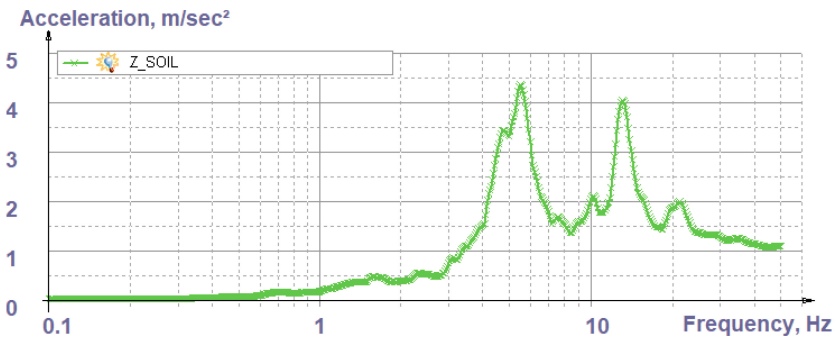


Fig. 10. Response spectra ($\xi = 0.05$) in Z direction

3.2 Configuration of the TMD

Figure 17 shows the configuration of the innovative TMD developed for the target building. The construction of the TMD consists from the building' technological floor and the roof that constitutes TMD's mass supported by the BCS 3D springs and 3D dampers system dampers provides close to optimum damping ratio $\xi_{TMD} = 0.1$. For the considered building according to the optimization analysis it is necessary to use 50 spring blocks with vertical stiffness $K_v = 1021$ t/m and horizontal stiffness $K_h = 298$ t/m and 25 3D dampers VD 426/219-7 by process conditions [26]. With these close to optimal TMD's parameters (mass, stiffness, damping) TMD provides to the building efficient protection both from wind and seismic loads with a quite reasonable relative displacements between TMD and the main structure from 20 to 45 mm only, Fig. 18.

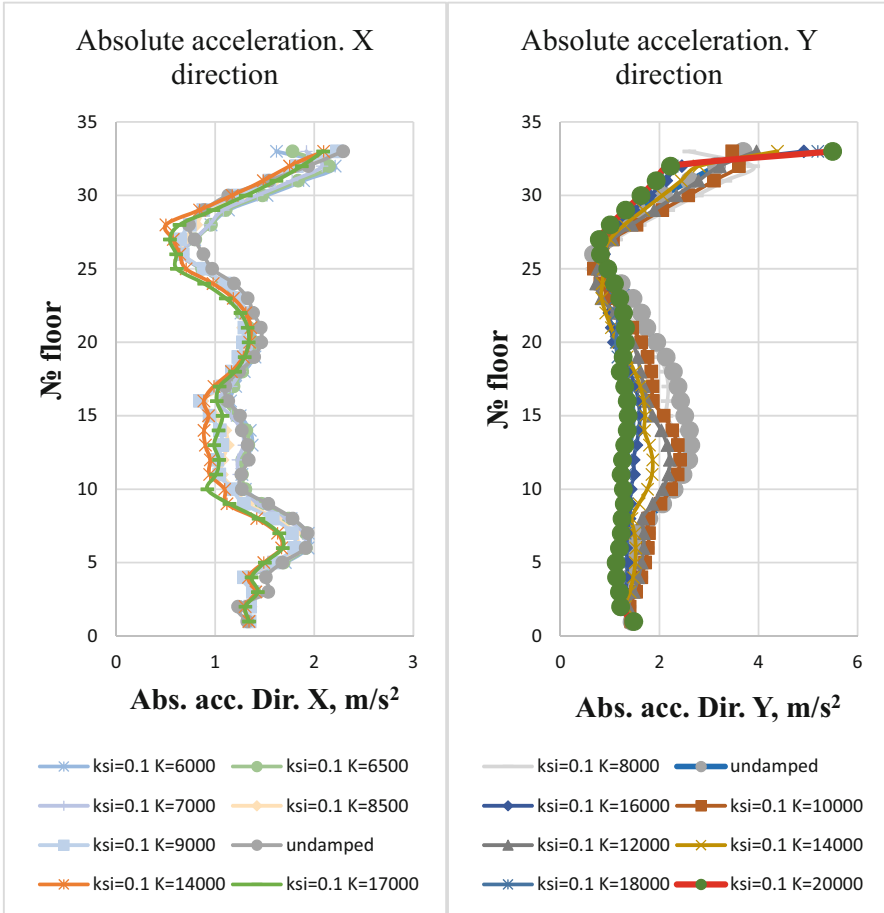


Fig. 11. Maximum storey acceleration in X, Y directions

3.3 Comparison of Analytical and Finite Elemental Models (FEM). Shear Force and Bending Moment in the Most Loaded Column

Program code of the analytical model mentioned below was written by means of Math-Cad's software. 3D FEM was used in order to assess stresses in building's elements. 3D building was created by SCad office's software. Figure 19 demonstrates the 3D FEM of target building. It has 153000 joints and 174400 finite elements. Seismic excitation was presented by kinematic time-dependent displacements of soil. The dissipation matrix for column and beam finite elements is formed by the coefficient of internal inelastic resistance of the material [29]. For beams and columns damping ratio was used as $\xi = 0.05$, for soil $\xi_{SOIL} = 0.15$ and for TMD's constructions $\xi_{TMD} = 0.1$.

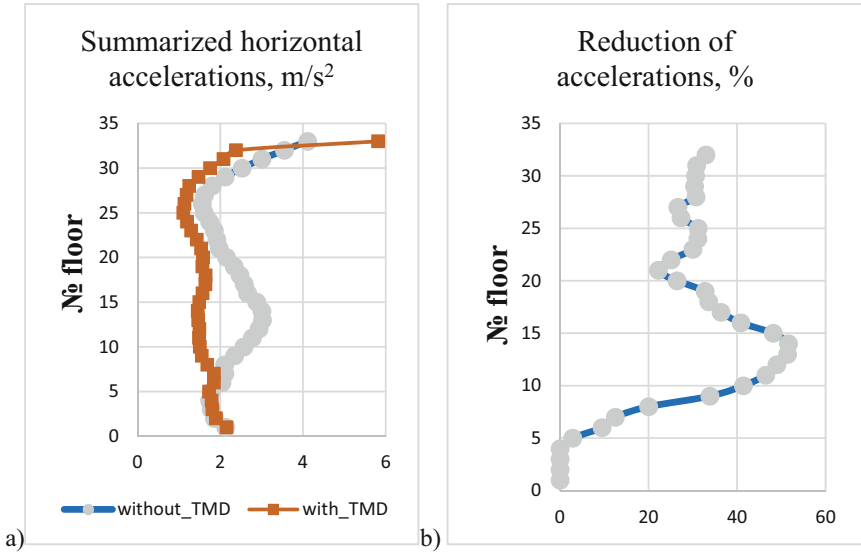


Fig. 12. a) Maximum summarized storey acceleration. b) Reduction of accelerations

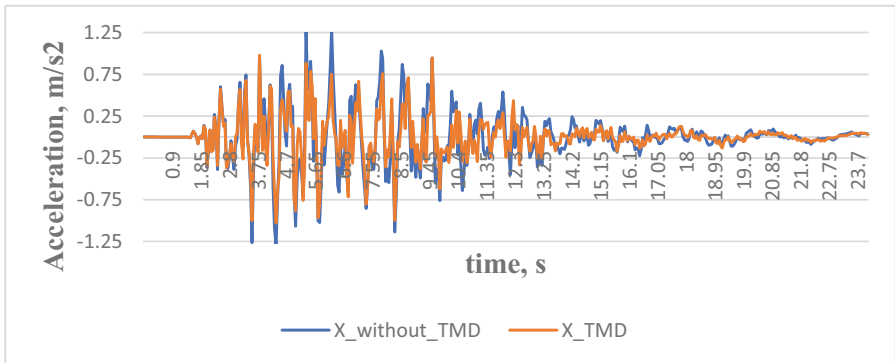


Fig. 13. Accelerations of the 13th floor in X direction

The comparison between analytical and FE models is shown in Fig. 20 and 21. Displacement of the last floor has good matching according to maximum and minimum values. Accelerations, in turn, has quite big difference because in analytical model is considered nondeformable slabs.

Analytical model allows in short time to define optimal parameters of TMD's constructions. After this analysis, it is reasonably to use FE model to assess stresses in building's elements. For example, in Fig. 22 and 23 is shown bending model and shear force in the most loaded column. Reduction of inner forces in column has reached up to 50%.

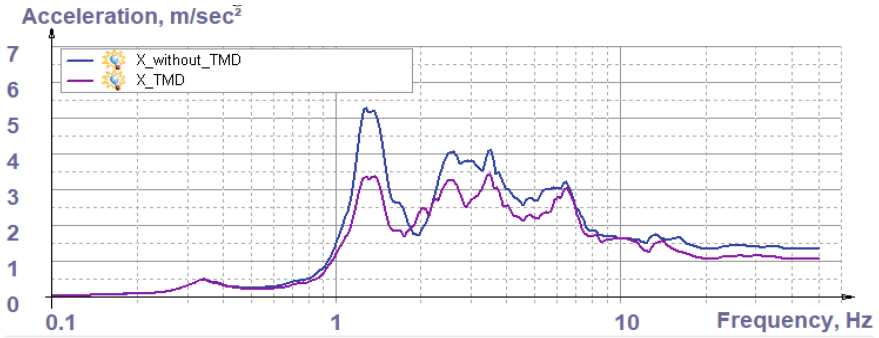


Fig. 14. Response spectra of the 13th floor ($\xi = 0.05$) in X direction

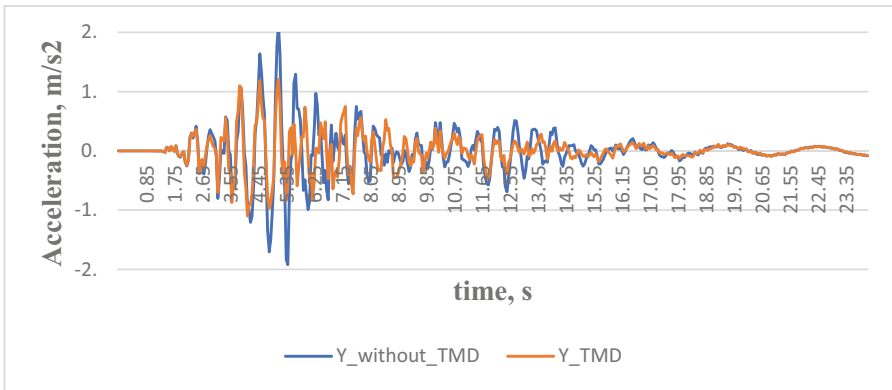


Fig. 15. Accelerations of the 13th floor in Y direction

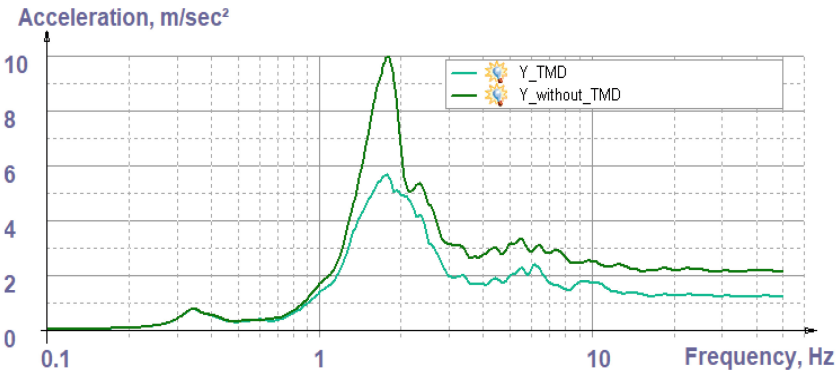
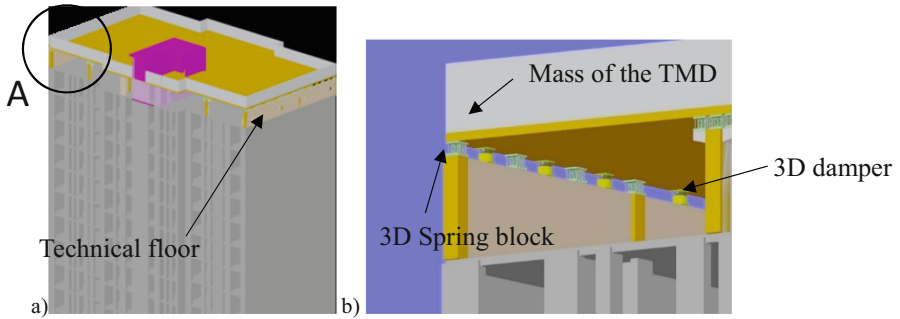
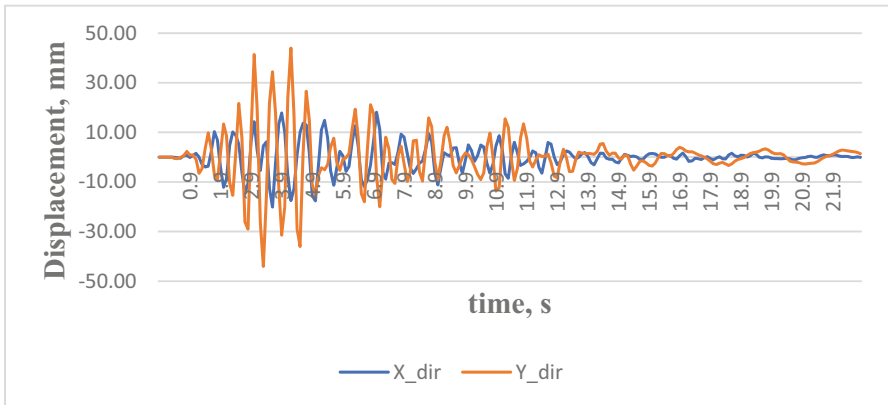


Fig. 16. Response spectra of the 13th floor ($\xi = 0.05$) in Y direction

Table 2. Eigen frequencies of dynamically controlled building by TMD

Number	Circular frequency, rad/s	Frequency, Hz	Direction	Modal mass, %
1	1.98	0.32	X	67.46
2	2.03	0.32	Y	64.85
3	10.3	1.64	X	7.54
4	11.1	1.77	Y	10.63
5	13.72	2.18	X	15.22
6	14.41	2.29	Y	11.18
7	33.18	5.28	X	8.73
8	34.69	5.52	Y	7.49

**Fig. 17.** a) General view of the building. b) Detailed view A**Fig. 18.** Relative displacements between TMD and primary building

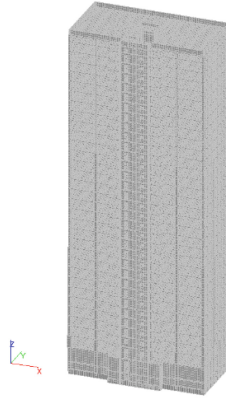


Fig. 19. 3D finite-element model of target building

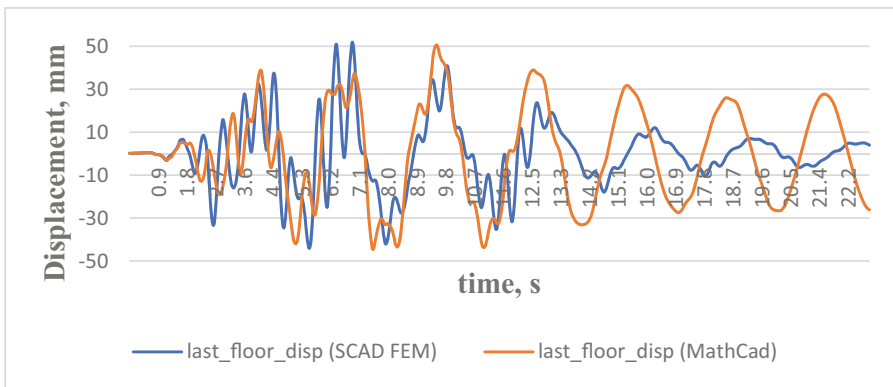


Fig. 20. Comparison between two models. Displacement

3.4 Reduction of the Vertical Accelerations in Target Building

Due to vertical stiffness of BCS spring blocks and using 3D FE model, it is possible to assess the efficiency of the TMD in Z-direction subjected to vertical seismic excitation (Fig. 7). Figures 24 and 25 show time-history acceleration and response spectra of the point which is close to shear wall of the building. TMD can reduce vertical acceleration up to 30% in comparison with uncontrolled building. It was achieved principally new effect of efficiency of the TMD in Z-direction. This effect has not been mentioned in recent investigations in TMD's field by others researchers [4–6, 12, 20–22].

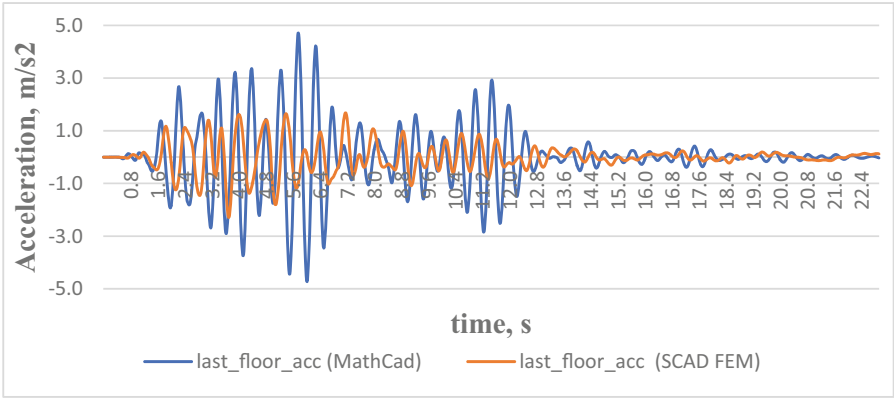


Fig. 21. Comparison between two models. Acceleration

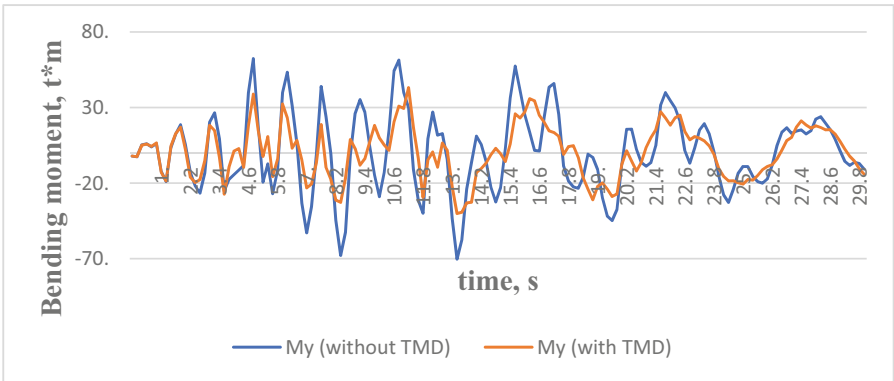


Fig. 22. Bending moment in column in target building with and without TMD

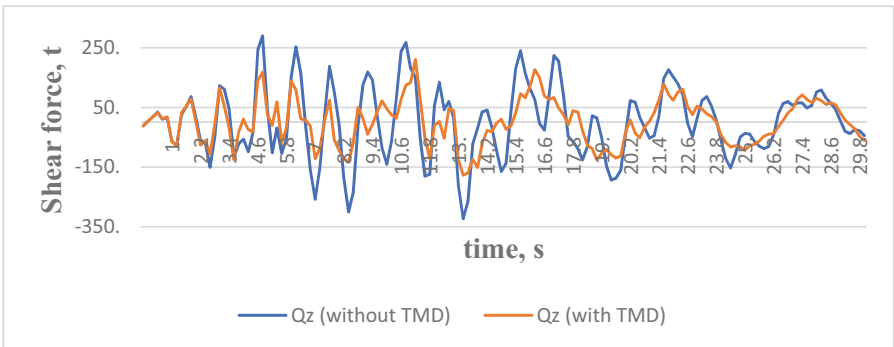


Fig. 23. Shear force in column in target building with and without TMD

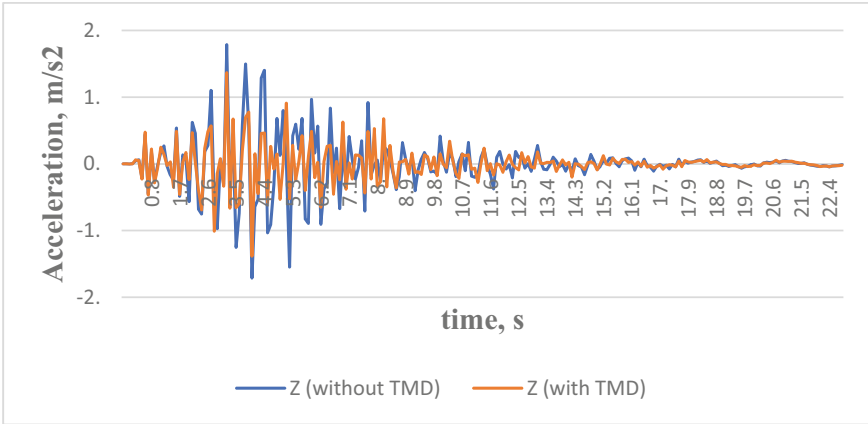


Fig. 24. Vertical absolute acceleration of the last floor in target building with and without TMD

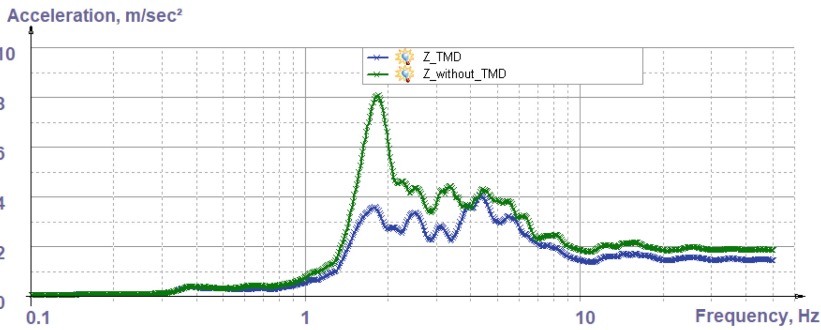


Fig. 25. Response spectra ($\xi = 0.05$) of the vertical absolute acceleration of the last floor with and without TMD

4 Conclusions

The main idea of proposed TMD is to use the existing upper technological part of the building located above the residential floors as the TMD device. This TMD is supported by the BCS (Base Control System) consists from 3D coil springs' supports and separately installed 3D viscodampers (VDs) [27, 28]. This new approach allows achieving significant improving in TMD's efficiency using optimal TMD's mass, stiffness and damping properties.

Proposed construction of the TMD has an optimal mass ratio of about 2%. It can be installed in existing buildings or in new buildings by using the upper technological floor and the roof as mass of a TMD. This solution doesn't require transportation of the huge TMD to the installation place at the upper floor of the structure. TMD is a passive seismic and wind control device and it doesn't require any external sources of energy or its maintenance. Relative displacements between the TMD and the building are quite appropriate, less than 5 cm, due to an optimal system's damping.

The matrix of stiffness was also developed. It allows to consider soil stiffness, bending and shifting modes of a building. It is very simple to use damping ratio in analysis separately for soil and for the TMD in case when TMD tuned not to the 1st eigen frequency of a building.

The approach of the usage the upper part of a building as a TMD can significantly reduce response accelerations and stresses in elements of the building along the entire height subjected to seismic and wind loads. It was shown that reduction of accelerations in comparison with dynamically uncontrolled building is up to 50%.

Developed TMD construction can reduce vertical acceleration and motion in primary building due to large mass of the roof. It was shown that vertical acceleration of the last floor was reduced up to 30% in comparison with dynamically uncontrolled building.

Innovative tuned mass dampers (TMD) were developed for reduction of seismic vibrations in high-rise buildings. But it also possible to use this construction of TDM for reduction of wind vibrations.

In general, TMD mentioned in this paper is able to reduce rocking and torsional motions caused by an earthquake and wind loads and these investigations are ongoing.

References

1. Frahm, H.: Device for damping vibrations of bodies. U.S. Patent 0989958 (1909)
2. Gutierrez Soto, M., Adeli, H.: Tuned mass dampers. In: Archives of Computational Methods in Engineering, vol. 20, no. 4 (2013). <https://doi.org/10.1007/s11831-013-9091-7>
3. Stanikzai, M.H., Elias, S., Matsagar, V.A., Jain, A.K.: Seismic response control of base-isolated buildings using multiple tuned mass dampers. *Struct. Des. Tall Spec. Build.* **28**(3) (2019). <https://doi.org/10.1002/tal.1576>
4. Salimi, M., Kamgar, R., Heidarzadeh, H.: An evaluation of the advantages of friction TMD over conventional TMD. *Innov. Infrastruct. Solut.* **6**(2) (2021). <https://doi.org/10.1007/s41062-021-00473-5>
5. Anajafi, H., Medina, R.A.: Comparison of the seismic performance of a partial mass isolation technique with conventional TMD and base-isolation systems under broad-band and narrow-band excitations. *Eng. Struct.* **158** (2018). <https://doi.org/10.1016/j.engstruct.2017.12.018>
6. de Barros, R.C., Sousa, C.M.: Pendulum TMD's in building vibration control. In: Garrido, P., Soares, F., Moreira, A. (eds.) *CONTROLO 2016*. LNEE, vol. 402, pp. 559–569. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-43671-5_47
7. Farghaly, A.A., Salem Ahmed, M.: Optimum design of TMD system for tall buildings. *ISRN Civ. Eng.* **2012** (2012). <https://doi.org/10.5402/2012/7164693>
8. Mondal, J., Nimmala, H., Abdulla, S., et al.: Tuned liquid damper. In: Proceedings of the 3rd International Conference on Mechanical Engineering and Mechatronics, no. 68, Prague, Czech Republic, August 2014
9. Momtaz, A.A., Abdollahian, M.A., Farshidianfar, A.: Study of wind-induced vibrations in tall buildings with tuned mass dampers taking into account vortices effects. *Int. J. Adv. Struct. Eng.* **9**(4) (2017). <https://doi.org/10.1007/s40091-017-0174-9>
10. Belver, A.V., Magdaleno, Á., Brownjohn, J.M.W., Lorenzana, A.: Performance of a TMD to mitigate wind-induced interference effects between two industrial chimneys. *Actuators* **10**(1) (2021). <https://doi.org/10.3390/act10010012>
11. Tian, L., Zeng, Y.: Parametric study of tuned mass dampers for long span transmission tower-line system under wind loads. *Shock Vib.* **2016** (2016). <https://doi.org/10.1155/2016/4965056>

12. Nakai, T., Kurino, H., Yaguchi, T., Kano, N.: Control effect of large tuned mass damper used for seismic retrofitting of existing high-rise building. *Jpn. Archit. Rev.* **2**(3) (2019). <https://doi.org/10.1002/2475-8876.12100>
13. Barkhordari, M.S., Tehranizadeh, M.: Ranking passive seismic control systems by their effectiveness in reducing responses of high-rise buildings with concrete shear walls using multiple-criteria decision making. *Int. J. Eng. Trans. B Appl.* **33**(8) (2020). <https://doi.org/10.5829/ije.2020.33.08b.06>
14. Li, J., Zhang, H., Chen, S., Zhu, D.: Optimization and sensitivity of TMD parameters for mitigating bridge maximum vibration response under moving forces. *Structures* **28** (2020). <https://doi.org/10.1016/j.istruc.2020.08.065>
15. Christopoulos, C., Filiatrault, A.: Principles of passive supplemental damping and seismic isolation. In: IUSS Press, vol. 133, no. 8 (2006)
16. Den Hartog, J.P.: *Mechanical Vibrations*. McGraw-Hill Book Company, The Maple Press Company, Ork (1934)
17. Falcon, K.S., Stone, B.J., Simcock, W.D., Andrew, S.: Optimization of vibration absorbers: a graphical method for use on idealized systems with restricted damping. *J. Mech. Eng. Sci.* **9**, 374–381 (1967)
18. Luft, R.W.: Optimal tuned mass dampers for buildings. *J. Struct. Div.* **105**(12), 2766–2772 (1979)
19. Warburton, G.B.: Optimal absorber parameters for various combinations of response and excitation parameters. *Earthq. Eng. Struct. Dyn.* **10**, 381–401 (1982)
20. Liu, Y., Wang, K., Mercan, O., Chen, H., Tan, P.: Experimental and numerical studies on the optimal design of tuned mass dampers for vibration control of high-rise structures. *Eng. Struct.* **211** (2020). <https://doi.org/10.1016/j.engstruct.2020.110486>
21. Lara-Valencia, L.A., Farbiarz-Farbiarz, Y., Valencia-González, Y.: Design of a tuned mass damper inerter (TMDI) based on an exhaustive search optimization for structural control of buildings under seismic excitations. *Shock Vib.* **2020** (2020). <https://doi.org/10.1155/2020/8875268>
22. He, H., Wang, W., Xu, H.: Multidimensional seismic control by tuned mass damper with poles and torsional pendulums. *Shock Vib.* **2017** (2017). <https://doi.org/10.1155/2017/5834760>
23. Carlot, J.: Effects of a tuned mass damper on wind-induced motions in tall buildings. master of engineering in civil and environmental engineering at the Massachusetts Institute of Technology (2012)
24. Birbraer, A.N.: *Seismic Analysis of Structures*. Nauka, St. Petersburg (1998), 255 p.
25. Chopra, A.K.: *Dynamics of structures 5th edition SI units* Anul K Chopra. *J. Chem. Inf. Model.* **53**(9) (2012)
26. TU 4192-001-20503039-01: Viscoelastic dampers VD series. Specifications. CKTI-Vibroseism, Saint Petersburg (1991/2022)
27. Kostarev, V., Nawrotzki, P., Vasil'ev, P., Vaindrakh, M.: Developing and natural scale testing of the 3D BCS base isolation system. In: *World Conference on Seismic Isolation, Energy Dissipation and Active Vibration Control of Structures Proceedings* (2021). <https://doi.org/10.37153/2686-7974-2019-16-460-472>
28. Nawrotzki, P., Kostarev, V., Siepe, D., Barutzki, F., Salcedo, V.: 3D seismic isolation systems for the nuclear industry layout, design & qualification. *Transactions, SMiRT-26 Berlin/Potsdam, Germany, 10–15 July 2022, Division V* (2022)
29. Fialko, S., Karpilovskyi, V.: Time history analysis formulation in SCAD FEA software. *J. Meas. Eng.* **6**(4) (2018). <https://doi.org/10.21595/jme.2018.20408>