

# BUCKLING OF THE PRESSURE BALANCED UNIVERSAL EXPANSION JOINTS UNDER INTERNAL PRESSURE

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## ABSTRACT

This paper considers a real case study of the bellows instability (squirm) observed in a pressure balanced universal expansion joint within a power plant during hydrostatic tests conducted immediately after piping installation. The investigation aims to identify the root causes of this instability by considering potential modes of failure in the universal expansion joint. The analysis takes into consideration the impact of actual piping stiffness on critical pressure, along with additional lateral stiffness occurred when the tie rods of the expansion joint are inclined. The critical pressure values obtained through this study are then compared against stability requirements outlined in EJMA standard.

## INTRODUCTION

Expansion Joints are widely implemented in the design of piping systems for thermal and nuclear power plants. The use of these devices allows to implement a more compact layout of piping and reduce the load on the sensitive equipment nozzles maintaining the operability and integrity of the piping system. As a general rule, the assessment of expansion joints is carried out by the manufacturer by means of fatigue testing. At the same time, in the frame of piping flexibility and stress analysis an engineer should ensure that expansion joints movements are complied with permissible values set by manufacturer. However, operating experience shows that this approach does not always allow to exclude such a failure mode of expansion joints as a column buckling due to internal pressure. This can be explained by the fact that, by default, the manufacturer determines the value of the critical buckling pressure according to the equations of the EJMA (EJMA 2015) and/or EN standards, implying that installation procedure for expansion joints (i.e. number and type of piping supports around expansion joint) is strictly following to these guides.

One potential failure mode for expansion joints is the loss of stability induced by internal pressure. This pressure can trigger two distinct buckling modes: in-plane squirm and column squirm. In the instance of in-plane squirm, the bellows pitches undergo a wave-like pattern around the circumference after significant development of the yield regions. On the other hand, column squirm manifests within a range characterized by a relatively large number of convolutions. In this scenario, the bellows buckle like a column subjected to an axial compression load. The EJMA standard offers simplified design rules that impose limitations on both in-plane and column squirm.

The critical pressure for the column instability of the expansion joint significantly depends on the boundary conditions applied to its ends. EJMA establishes design pressure limits for column squirm based on four basic types of end conditions. However, in practice, the expansion joint manufacturer typically calculates the design pressure limits for column squirm using the EJMA formula, specifically for the scenario involving rigidly supported ends of the expansion joint. Additionally, manufacturers commonly conduct hydrostatic tests on expansion joints with fixed ends. With this design approach, it becomes challenging to entirely prevent instances of instability in the expansion joint due to internal pressure if the stiffness of the connected piping segments proves to be insufficient. Additionally, it is important to highlight that piping engineering software typically does not address the stability concerns of piping systems equipped with expansion joints. As a result, potential design flaws related to inadequate rigidity of the piping near the expansion joints may go unnoticed.

## CASE STUDY

As an illustration, this paper considers a real incident occurred at a power plant. Universal pressure-balanced expansion joints with a diameter of DN600 were incorporated into the design of the heating water piping. The objective was to reduce temperature loads on the pump nozzles, ensuring that they remained within acceptable values. The operational parameters for this piping system are as follows:

$p = 2.2$  MPa,  $t = 115$  °C, the hydrotest pressure is  $p_{test} = 3.125$  MPa. The overall configuration of this system is depicted in Figure 1.

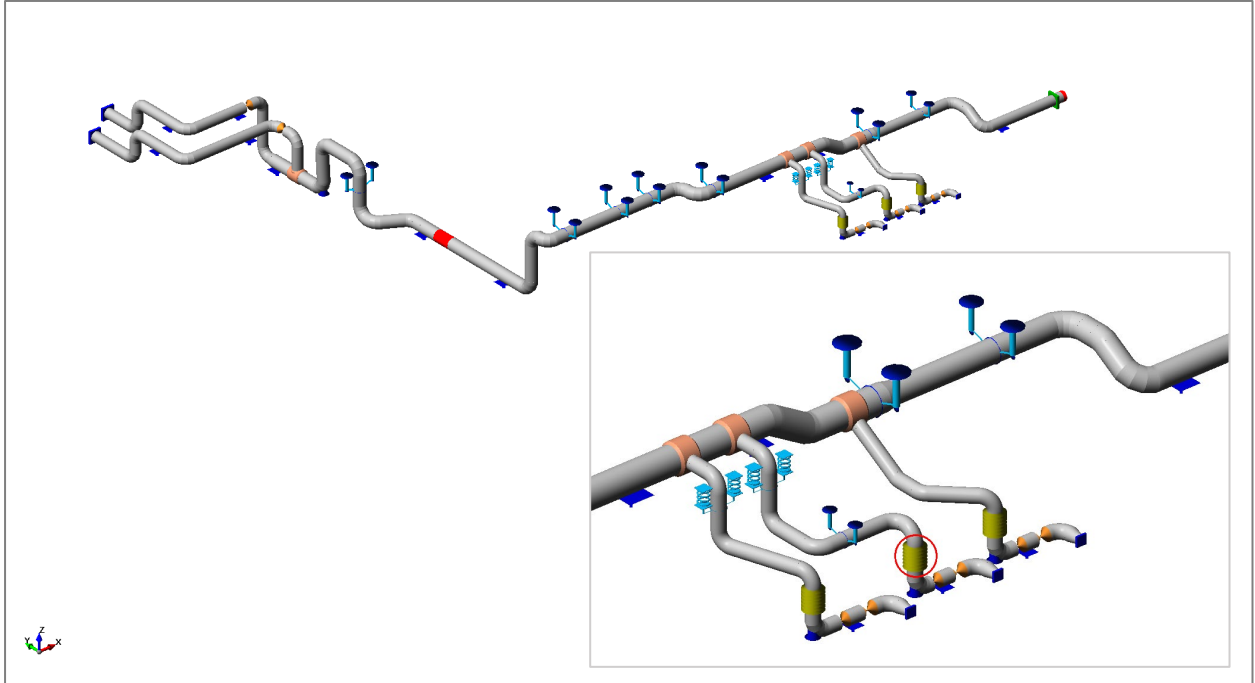


Figure 1. Piping layout

During the hydrostatic test, one of the three DN600 expansion joints failed due to instability. This expansion joint was located on the most flexible piping branch. The failure occurred at a temperature of  $t = 22$  °C and an internal pressure of  $p = 2.7$  MPa, Figure 2. Figure 3 shows the expansion joint before and after the test. After the pressure drop, the joint had a residual plastic deformation of 90 mm.

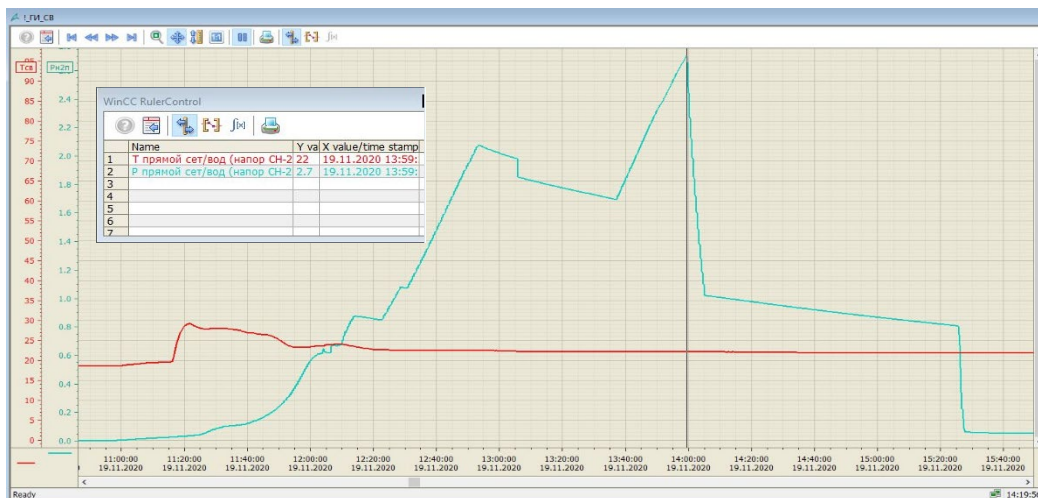


Figure 2. Pressure and temperature during hydrotest



Figure 3. Expansion joint before and after hydrotest

The failed device is a pressure balanced expansion joint with an additional bellows attached through an intermediate pipe. Such design provides compensation not only for axial but also for lateral deformation of the piping. The thrust force from the pressure is absorbed by the rods and the internal sleeve of the expansion joint. The design of the expansion joint is illustrated in Figure 4.

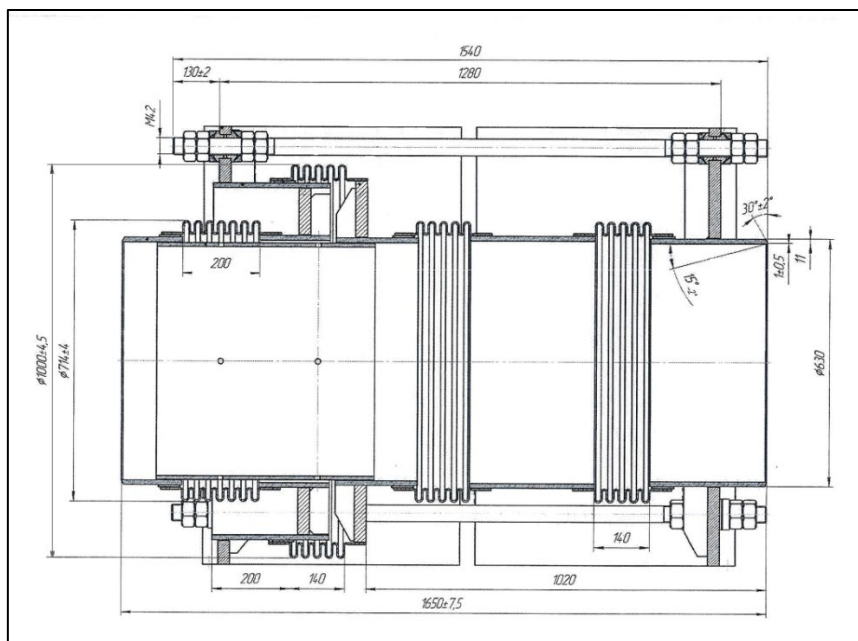


Figure 4. Design of the universal pressure balanced expansion joint

## ANALYSIS ACCORDING TO EJMA STANDARD

Taking into account the design of the expansion joint and the fact that the loss of stability occurred with the lateral displacement of the flanges, when determining the critical pressure, such an expansion joint can be considered as a conventional tied universal expansion joint, consisting of two bellows with an intermediate pipe (depicted as two bellows on the right in Figure 4). This approach allows for the application of EJMA equations to calculate the critical pressure.

The dimensions of the bellows under consideration are presented in Table 1 and illustrated in Figure 5.

Material of the bellows is steel 1.4541 (X6CrNiTi18-10) according to EN 10028-7:2008. Modulus of Elasticity of material at room temperature  $E_b = 1.95 \times 10^5$  MPa. Yield strength at room temperature in annealed condition from the certified test report  $S_{ym} = 244$  MPa.

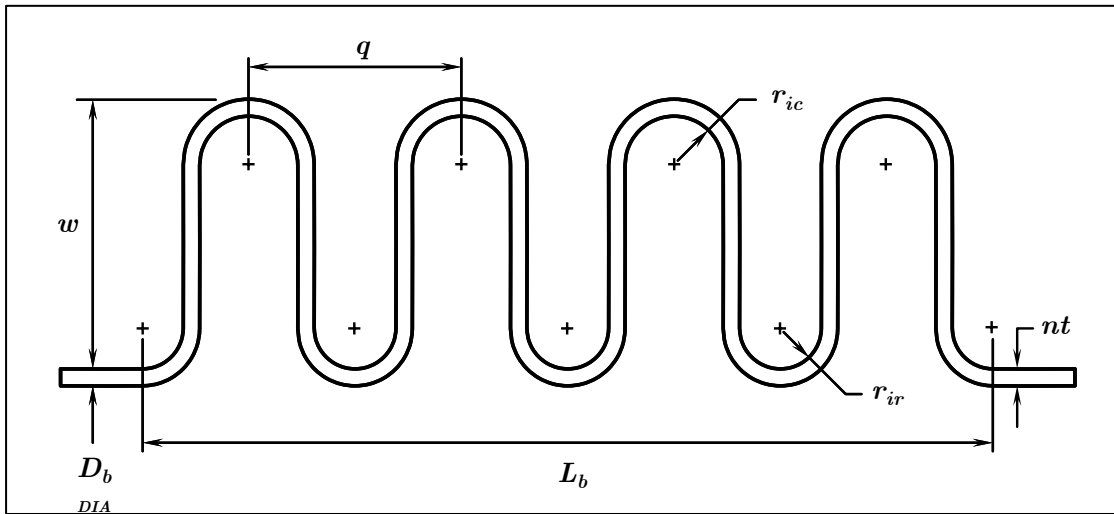


Figure 5. Bellows geometry

Table 1. Dimensions of the expansion joint

Number of convolutions in one bellows	$N = 5$
Inside diameter bellows convolutions	$D_b = 631$ mm
Number of bellows material plies	$n = 5$
Bellows nominal material thickness of one ply	$t = 0.8$ mm
Convolution pitch	$q = 30$ mm
Convolution height	$w = 37.5$ mm
Crest convolution inside radius	$r_{ic} = 6$ mm
Root convolution inside radius	$r_{ir} = 5$ mm
Bellows convoluted length	$L_b = Nq = 150$ mm
Mean diameter of bellows convolutions	$D_m = D_b + w + nt = 672.5$ mm
Mean radius of bellows convolution	$r_m = (r_{ic} + r_{ir} + nt)/2 = 7.5$ mm

The calculation of auxiliary quantities is based on the EJMA guidelines.

The yield strength at room temperature in the as-formed condition (with cold work) is:

$$S_y = 0.67C_m S_{ym} = 0.67 \times 3.0 \times 244 = 488 \text{ MPa.} \quad (1)$$

The bellows material thickness  $t_p$  for one ply, corrected for thinning during forming is calculated as:

$$t_p = t \sqrt{\frac{D_b}{D_m}} = 0.8 \sqrt{\frac{631}{672.5}} = 0.775 \text{ mm.} \quad (2)$$

Factors utilized in specific design calculations to correlate U-shaped bellows convolution segment behavior to that of a simple strip beam are:

$$C_p = 0.696, \quad C_f = 1.509.$$

Column instability pressure reduction factor based on imposed angular rotation is  $C_\theta = 1.0$ .

The theoretical axial elastic spring rate per convolution of the bellows is calculated as follows:

$$f_{iu} = 1.7 \frac{D_m E_b t_p^3 n}{w^3 C_f} = 1.7 \frac{672.5 \times 195000 \times 0.775^3 \times 5}{37.5^3 \times 1.509} = 6520 \text{ N/mm,} \quad (3)$$

subsequently, the theoretical axial elastic spring rate of the entire bellows can be determined as:

$$K_a = \frac{f_{iu}}{N} = \frac{6520}{5} = 1304 \text{ N/mm.} \quad (4)$$

At the operating pressure  $p = 2.7$  MPa the meridional bending stresses at the bellows will be equal:

$$S_4 = \frac{P}{2n} \left( \frac{w}{t_p} \right)^2 C_p = \frac{2.7}{2 \times 5} \left( \frac{37.5}{0.775} \right)^2 \times 0.696 = 440 \text{ MPa.} \quad (5)$$

It is important to highlight that this value is lower than the yield stress for the bellows material ( $S_4 = 440 \text{ MPa} < S_y = 488 \text{ MPa}$ ), indicating that an elastic behavior is anticipated. Consequently, the Euler equation can be employed for further evaluation.

The EJMA standard establishes the maximum allowable internal design pressure for single bellows, considering column instability, and utilizes a safety factor of 2.25. For universal expansion joints,  $P_{sc}$  is calculated as follows:

$$P_{sc} = \frac{0.34\pi C_\theta f_{iu}}{N^2 q} = \frac{0.34\pi \times 1.0 \times 6520}{10^2 \times 30} = 2.32 \text{ MPa.} \quad (6)$$

where  $N$  is a total number of convolutions in both bellows.

Equation (6) assume that each end of the expansion joint is rigidly supported (fixed). However, if one end is fixed and the other is laterally guided, the limiting design pressure  $P_{sc}$  is reduced to 0.25 times its original value.

In accordance with EJMA standard, the test pressure should not exceed 1.5 times the limiting design pressure  $P_{sc}$ . For the current scenario, the allowable test pressure for fixed boundary conditions exceeds the test pressure set in the piping design:  $1.5 \times 2.32 = 3.48 > 3.125$  MPa. On the other hand, for fixed-laterally guided conditions the maximum allowable test pressure becomes even less than the piping operating pressure:  $0.25 \times 3.48 = 0.87 < 2.7$  MPa.

These results highlight the need for a more precise solution, considering the actual boundary conditions.

## ANALYTICAL SOLUTION

Studies by Newland (1964) and Broyles (1989) have demonstrated a close analogy between the buckling problem of a bellows under internal pressure and the buckling of a compressed strut. The critical pressure of the bellows can be calculated using the well-known Euler formula, if instead of force and bending stiffness, the pressure thrust and the equivalent bending stiffness of the bellows are used.

Euler equation:

$$P_c = \frac{\pi^2 EI}{(\mu l)^2}. \quad (7)$$

Pressure trust:

$$P_c = \frac{\pi}{4} D_m^2 p_c, \quad (8)$$

Equivalent bending stiffness:

$$EI = \frac{1}{8} K_a D_m^2 l. \quad (9)$$

Substituting (8) and (9) into (7) one can derive the critical pressure equation in the form:

$$p_c = \frac{1}{2} \frac{\pi K_a}{\mu^2 l}, \quad (10)$$

where  $\mu$  is the so-called “length reduction factor”, the value of which depends on the boundary conditions of the bellows.

The calculation model of a tied universal expansion joint is shown in Fig. 6. Two elastic bellows of length  $l$  are connected to each other by a rigid intermediate pipe of length  $a$ . One end of the expansion joint is rigidly clamped, and an elastic spring with stiffness  $k_p$  is attached to the other end, simulating the stiffness of the connected piping. The thrust force from pressure  $P = \pi D_m^2 p / 4$  is accommodated by ties of length  $b$ . Due to the inclination of the tie rods caused by lateral displacement  $\delta$ , an additional lateral force  $P\delta/b$  arises, preventing this displacement. Total lateral force  $Q$  can be expressed as:

$$Q = k_p \delta + \frac{P\delta}{b} = k\delta, \text{ where } k = k_p + \frac{P}{b} \quad (11)$$

Let's write down the deflection equations for the left and right bellows:

$$EI y_1'' = M + P(\delta - y_1) - Q(L - x_1), \quad (12)$$

$$EI y_2'' = M + P(\delta - y_2) - Qx_2 \quad (13)$$

where:  $Q = k\delta$ ,  $k = k_p + \frac{P}{b}$  and  $L = 2l + a$ .

The solution will be found in the form:

$$y_1 = C_1 \sin(\alpha x_1) + C_2 \cos(\alpha x_1) + \frac{M}{P} + \left[ 1 - \frac{k}{P}(L - x_1) \right] \delta, \quad (14)$$

$$y_2 = C_3 \sin(\alpha x_2) + C_4 \cos(\alpha x_2) + \frac{M}{P} + \left( 1 - \frac{k}{P} x_2 \right) \delta, \quad (15)$$

where:  $C_1, C_2, C_3$  и  $C_4$  - arbitrary constants, and  $\alpha = \sqrt{P/EI}$ .

The solution will be found in the form:

$$y_1 = C_1 \sin(\alpha x_1) + C_2 \cos(\alpha x_1) + \frac{M}{P} + \left[ 1 - \frac{k}{P}(L - x_1) \right] \delta, \quad (16)$$

$$y_2 = C_3 \sin(\alpha x_2) + C_4 \cos(\alpha x_2) + \frac{M}{P} + \left( 1 - \frac{k}{P} x_2 \right) \delta, \quad (17)$$

where:  $C_1, C_2, C_3$  и  $C_4$  - arbitrary constants, and  $\alpha = \sqrt{P/EI}$ .

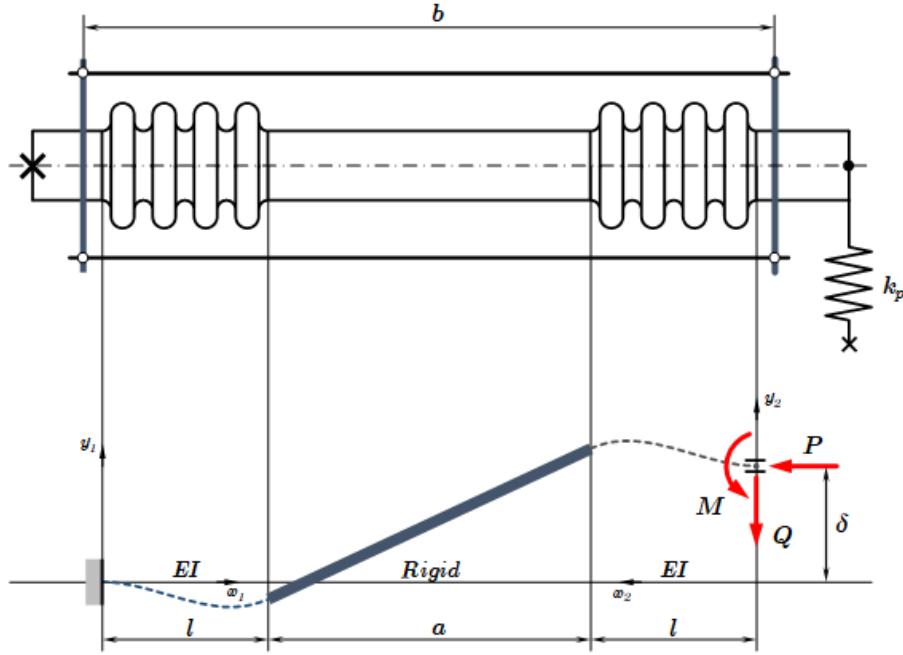


Figure 6. Model of tied expansion joint

The boundary conditions for these equations can be expressed as:

$$y_1(0) = 0, \quad (18)$$

$$y_1'(0) = 0, \quad (19)$$

$$y_2(0) = \delta, \quad (20)$$

$$y_2'(0) = 0, \quad (21)$$

$$y_1'(l) = -y_2'(l), \quad (22)$$

$$y_1(l) + y_1'(l)a = y_2(l). \quad (23)$$

From the boundary conditions (17), (18), (19) one can found that:

$$\delta = -\frac{P}{k}\alpha C_1, \quad \frac{M}{P} = -C_4, \quad C_3 = -C_1.$$

After substituting these values into (16), (20) and (21) we will derive the following system of equations:

$$\begin{aligned} C_1(2\sin(\alpha l) + \beta \cos(\alpha l)) + C_2(\cos(\alpha l) - \beta \sin(\alpha l)) - C_4 \cos(\alpha l) &= 0, \\ C_1\gamma + C_2 - C_4 &= 0, \\ (C_2 + C_4)\sin(\alpha l) &= 0, \end{aligned} \quad (24)$$

where:  $\beta = \alpha a$ ,  $\gamma = \alpha(L - P/k)$ .

A system of homogeneous equations yields a non-zero solution only when its determinant is equal to zero, therefore:

$$\begin{vmatrix} 2\sin(\alpha l) + \beta \cos(\alpha l) & \cos(\alpha l) - \beta \sin(\alpha l) & -\cos(\alpha l) \\ \gamma & 1 & -1 \\ 0 & \sin(\alpha l) & \sin(\alpha l) \end{vmatrix} = 0. \quad (25)$$

Expanding the determinant, we obtain the transcendental equation:

$$\sin(\alpha l) \left[ (4 + \beta\gamma) \sin(\alpha l) + 2(\beta - \gamma) \cos(\alpha l) \right] = 0, \quad (26)$$

from which we obtain two equations to determine the critical force:

$$\sin(\alpha l) = 0, \quad (27)$$

$$\tan(\alpha l) = \frac{2(\gamma - \beta)}{4 + \beta\gamma} = \frac{\left[ 4 \left( \frac{k_p^*}{(\alpha l)^2} + \frac{l}{b} \right) - 2 \right] (\alpha l)}{\left[ 4 + \left( 2 + \frac{a}{l} \right) \frac{a}{l} (\alpha l)^2 \right] \left( \frac{k_p^*}{(\alpha l)^2} + \frac{l}{b} \right) - \frac{a}{l} (\alpha l)^2} \quad (28)$$

where  $k_p^* = \frac{k_p l^3}{EI}$ .

Having calculated the smallest non-zero root  $\alpha l$  of these equations, from the definition

$$\alpha = \sqrt{P/EI} \text{ we find the critical force } P_c = \alpha^2 EI = \frac{\pi^2 EI}{(\mu l)^2}, \text{ where } \mu = \pi / \alpha l.$$

The smallest non-zero root of equation (25) is  $\alpha l = \pi$ . In this case, the critical force corresponds to the theoretical critical pressure of a conventional bellows of length  $2l$  with fixed ends. Loss of stability of the expansion joint in this case occurs without lateral displacement in the form of Mode 1, shown in Figure 7.

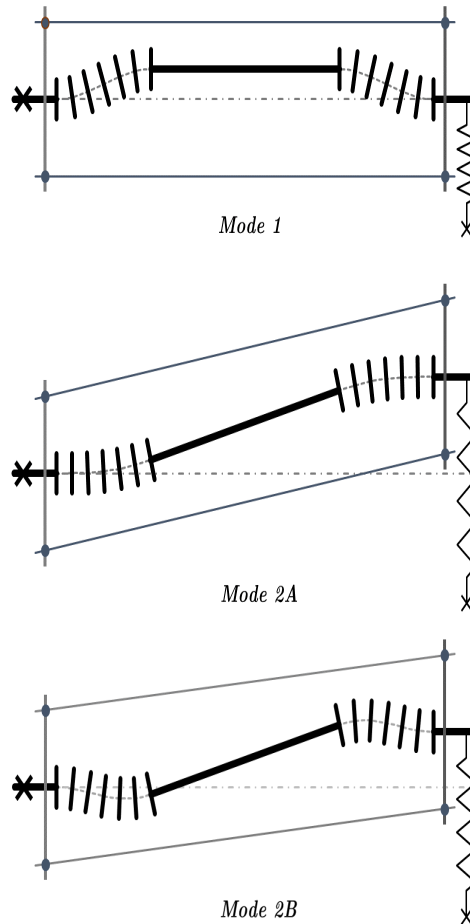


Figure 7. Buckling modes of tied universal expansion joint



The smallest non-zero root of equation (26) is determined numerically, corresponding to a buckling mode with lateral displacement. Depending on the lateral stiffness, the shape of the elastic line can exhibit either an inflection point (Mode 2B) or lack thereof (Mode 2A).

Figure 8 shows the dependence of the dimensionless critical forces  $(\alpha l)^2 = P_c l^2 / EI$ , obtained from equations (25) and (26), depending on the dimensionless stiffness  $k_p / k_v$ , where

$$k_v = \frac{3}{4} \frac{K_a D_m^2}{l^2 + 3(a+l)^2} - \text{lateral stiffness of the expansion joint. The graphs were plotted for different}$$

lengths of the connected pipe ( $a/l = 0, 1, 2, 4$ ), the length of the ties was taken equal to  $b = 2l + a$ . It can be seen that with low piping stiffness, the expansion joint buckles first in Mode 2, and the critical load can be significantly lower than for a bellows with fixed ends. With increasing length of the tie rods, the load reduction will be even greater.

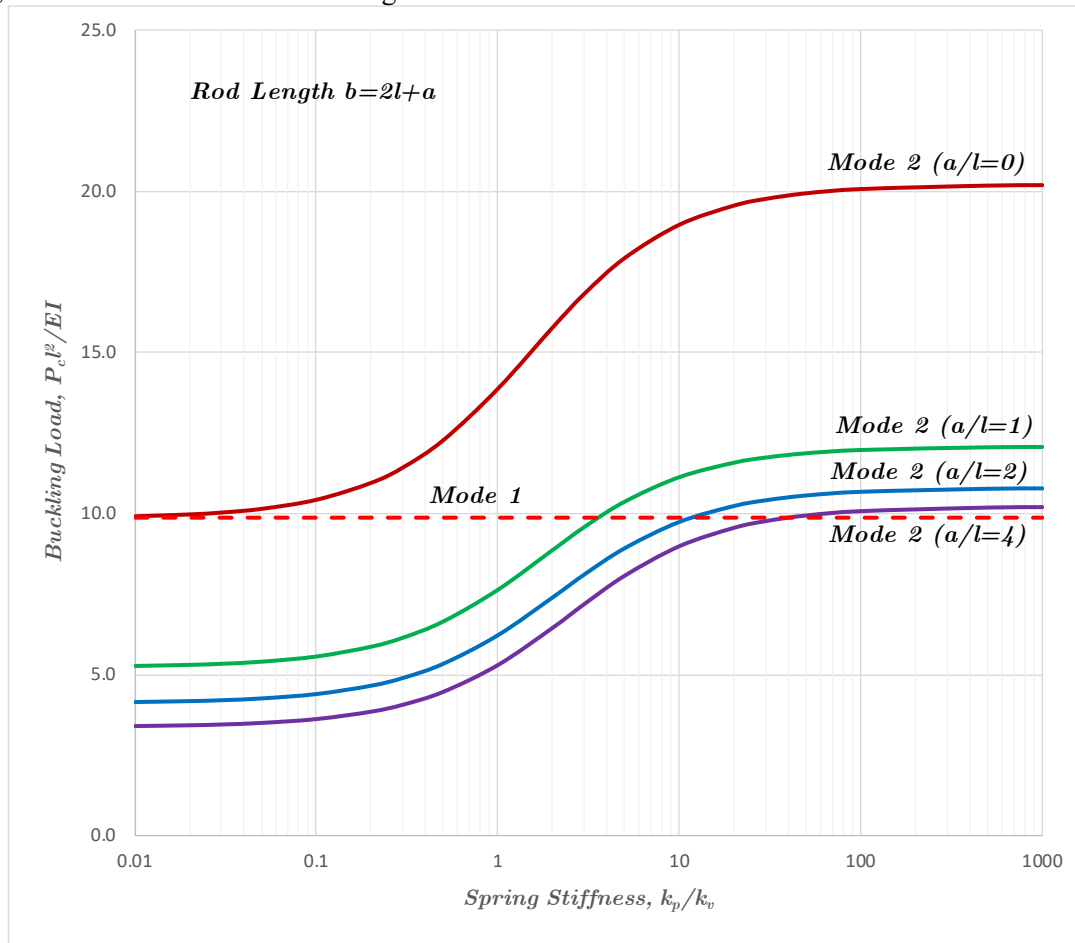


Figure 8. Dependence of buckling load  $(\alpha l)^2 = P_c l^2 / EI$  on stiffness ratio  $k_p / k_v$

Now, let's determine the critical buckling pressure of the universal expansion joint under consideration using the derived analytical solution. The input data and calculation of the axial stiffness of the bellows are provided in the previous section. Here, we present the additional data for the complete expansion joint: the length of the bellows  $l$  is 150 mm, the length of the intermediate pipe  $a$  is 300 mm, and the length of the tie rods  $b$  is 1280 mm. The equivalent stiffness of the bellows is:

$$EI = \frac{1}{8} K_a D_m^2 l = \frac{1304 \times 672.5^2 \times 150}{8} = 1.106 \times 10^{10} \text{ N} \cdot \text{mm}^2. \quad (29)$$

Lateral shear stiffness of the expansion joint calculated as:

$$k_v = \frac{3}{4} \frac{K_a D_m^2}{l^2 + 3(a+l)^2} = \frac{3}{4} \frac{1304 \times 672.5^2}{150^2 + 3 \times 450^2} = 702 \text{ N/mm}^2. \quad (30)$$

The minimum stiffness of the piping  $k_p$  was calculated using the dPIPE software, dPIPE 2017. To do this, the lateral stiffness of the expansion joint was set equal to 0, and its ends were shifted relative to each other in a given direction by 1 mm. The resulting dependence of the piping stiffness on the direction is shown in Figure 9. The calculated minimum lateral stiffness of the piping  $k_p = 513 \text{ N/mm}$  turned out to be less than the lateral stiffness of the expansion joint itself ( $k_p/k_v = 513/702 = 0.73$ ).

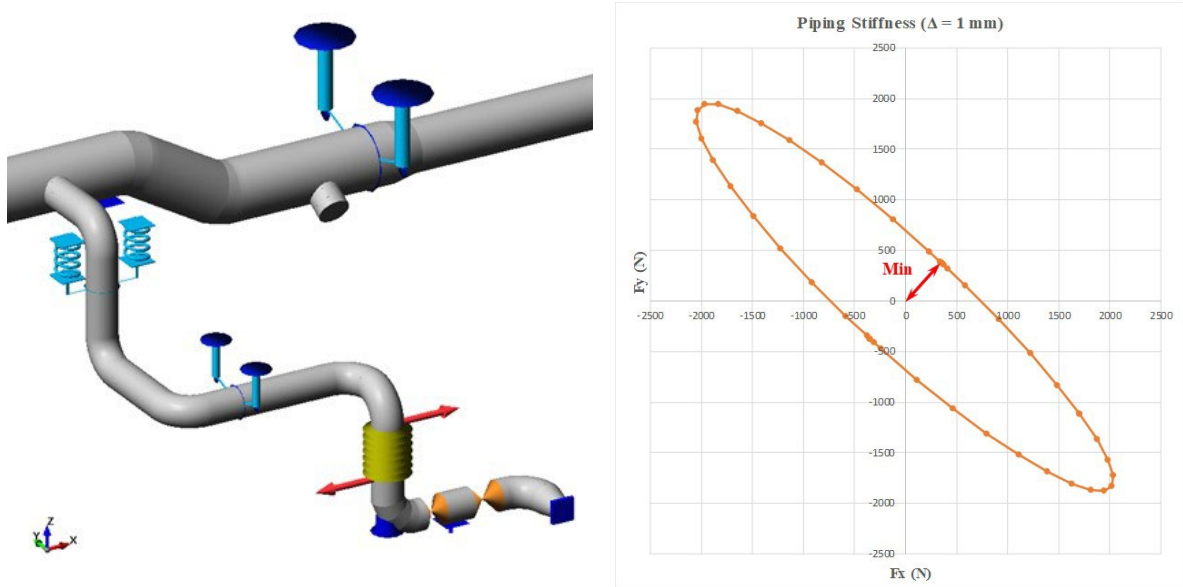


Figure 9. Identifying the most flexible direction of the piping

Calculating the length reduction factor from equation (26) and the critical pressure according to formula (10), we obtain:

$$\mu = 2.166,$$

$$p_c = \frac{1}{2} \frac{\pi K_a}{\mu^2 l} = \frac{\pi \times 1304}{2 \times 2.166^2 \times 150} = 2.91 \text{ MPa}.$$

The critical pressure determined by above analysis closely aligned with the actual buckling pressure of the expansion joint (2.91 MPa vs 2.7 MPa). This alignment suggests that the proposed calculation procedure accurately represents the posed problem and can be effectively applied to address similar challenges.

## CONCLUSIONS

- 1) This study investigates a real incident, employing a dual approach by reviewing recommendations from design guides like EJMA and obtaining a numerical analytical solution to understand the failure's root causes.
- 2) The EJMA recommendations for the critical pressure of installed expansion joint are highly sensitive to the bellows boundary conditions.
- 3) The analytical solution, based on the actual piping stiffness, gives a critical pressure that aligns well with the recorded value at the plant.

- 4) Expansion Joints Design Guides and Manufacturer's Catalogues offer precise recommendations for proper device installation, ensuring that adherence to these guidelines can prevent Expansion Joints from failing. In the specific case discussed, the introduction of additional guide support on the horizontal section above the bellows could significantly enhance its stability. Using formula (10), a critical pressure of  $p_c = 13.65$  MPa could be achieved (in this case  $\mu = 1$ ). Even considering the standard margin of 1.5 according to the EJMA standard, the permissible pressure based on stability conditions during a hydrostatic test would be 9.1 MPa, substantially surpassing the design hydraulic pressure in the system.
- 5) Designers of piping systems should not rely solely on piping software: one should keep in mind, that conventional programs do not provide assessment for a possible expansion joint's buckling. Therefore, the manufacturer's recommendations and design guides are imperative to be followed to ensure a safe and reliable installation.

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